Math 32B, Lecture 4
Multivariable Calculus

Sample Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books, notes, or calculators. Show all your work; partial credit will be given for progress toward correct solutions, but unsupported correct answers will not receive credit. Remember to make your drawings large and clear, and to label your axes.

Write your solutions in the space below the questions. If you need more space, use the back of the page. Do not turn in your scratch paper.

Name: Solutions

UID: ______

Section: ______

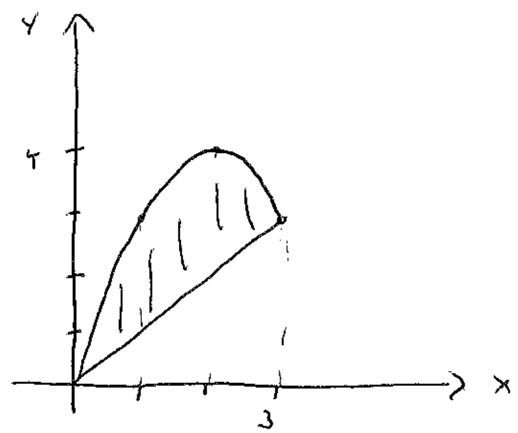
Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] Draw the region in the plane bounded by $y = x$ and $y = 4x - x^2$.

(b) [5pts.] Compute the double integral of the function $f(x, y) = x$ over this region.

(a)



(b)

$$\int_0^3 \int_x^{4x-x^2} x \, dy \, dx = \int_0^3 x\left(4x - x^2 - x\right) dx$$

$$= \int_0^3 \left(4x^2 - x^3 - x^2\right) dx$$

$$= \int_0^3 \left(3x^2 - x^3\right) dx$$

$$= \left[x^3 - \frac{1}{4}x^4\right]_0^3$$

$$= 27 - \frac{81}{4}$$

$$= \frac{27}{4}$$

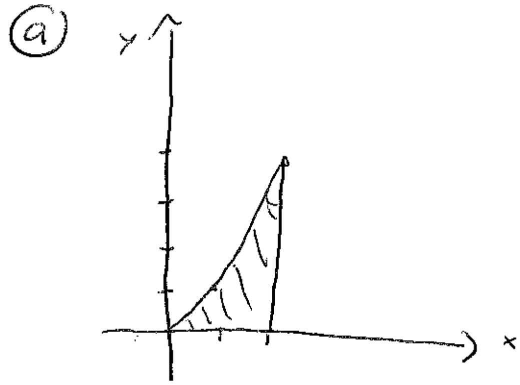
Problem 2.

Evaluate the integrals. Use any method you like.

(a) [5pts.] $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\,dx\,dy$

(b) [5pts.] $\int_0^1 \int_0^3 \int_0^{\sqrt{9-x^2}} z\tan^{-1}(\frac{y}{x})\,dy\,dx\,dz$

(a)



Change the order.

$$\int_0^2 \int_0^{x^2} \sqrt{x^3+1}\,dy\,dx$$

$$= \int_0^2 x^2\sqrt{x^3+1}\,dx$$

$$= \frac{1}{3}\,\frac{2}{3}\,(x^3+1)^{3/2}\Big|_0^2$$

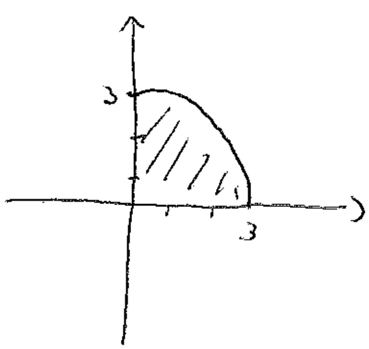
$$= \frac{2}{9}\left[9^{3/2} - 1^{3/2}\right]$$

$$= \frac{2}{9}\left[27-1\right]$$

$$= \frac{52}{9}$$

(b) Change to cylindrical

xy - region



$$\int_0^1 \int_0^{\pi/2} \int_0^3 z\,\theta\,r\,dr\,d\theta\,dz$$

$$= \int_0^1 z\,dz \cdot \int_0^{\pi/2} \theta\,d\theta \cdot \int_0^3 r\,dr$$

$$= \left(\frac{1}{2}z^2\Big|_0^1\right)\cdot\left(\frac{1}{2}\theta^2\Big|_0^{\pi/2}\right)\cdot\left(\frac{1}{2}r^2\Big|_0^3\right)$$

$$= \frac{1}{2}\cdot\left(\frac{1}{2}\left(\frac{\pi^2}{4}\right)\right)\cdot\left(\frac{1}{2}(9)\right)$$

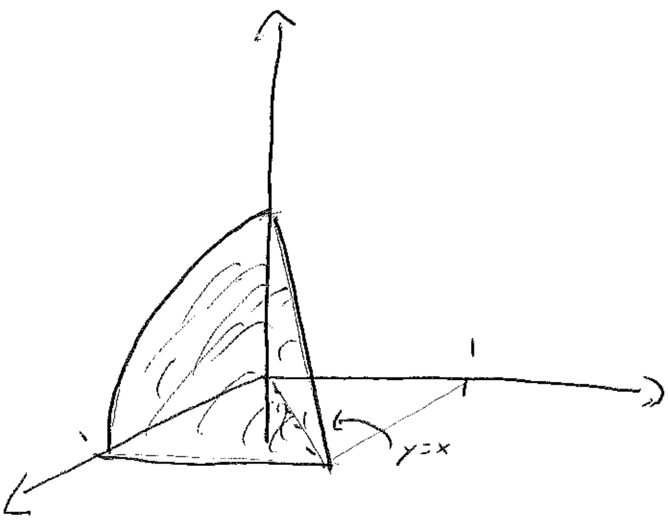
$$= \frac{9\pi^2}{32}$$

Problem 3.

(a) [5pts.] Draw the region in the first octant bounded by $z = 1 - x^2$ and $y = x$.

(b) [5pts.] Write down (but do not evaluate) three different triple integrals that compute the volume of this region.

(a)



(b) e.g. $\int_0^1 \int_0^x \int_0^{1-x^2} dz\,dy\,dx$

$\int_0^1 \int_0^{1-x^2} \int_0^x dy\,dz\,dx$

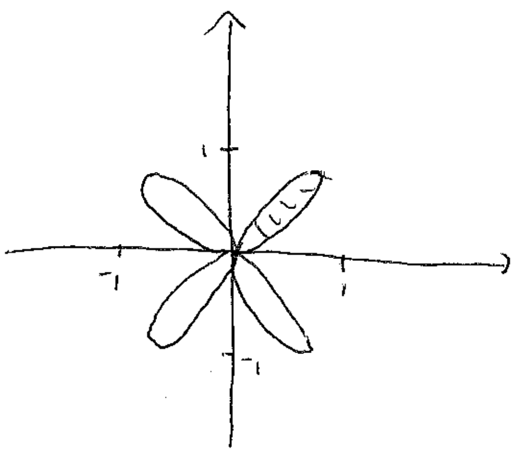
$\int_0^1 \int_0^{\sqrt{1-z}} \int_0^x dy\,dx\,dz$

Problem 4.

(a) [5pts.] Draw the polar curve $r = \sin(2\theta)$.

(b) [5pts.] Find the area inside the portion of this curve lying in the first quadrant.

(a)



(b) $$\int_0^{\pi/2}\int_0^{\sin(2\theta)} r\,dr\,d\theta = \int_0^{\pi/2} \tfrac{1}{2}r^2\Big|_0^{\sin(2\theta)} d\theta$$

$$= \int_0^{\pi/2} \tfrac{1}{2}\sin^2(2\theta)\,d\theta$$

$$= \int_0^{\pi/2} \tfrac{1}{2}\left(\tfrac{1}{2}(1-\cos(2\theta))\right)d\theta$$

$$= \tfrac{1}{4}\left[\theta - \tfrac{1}{2}\sin 2\theta\right]_0^{\pi/2}$$

$$= \tfrac{1}{4}\left[\tfrac{\pi}{2}\right]$$

$$= \tfrac{\pi}{8}$$

Problem 5.

(a) [5pts.] Convert the equation $x^2 + y^2 + z^2 \leq 2z$ into an equation in spherical coordinates. Simplify as much as you can.

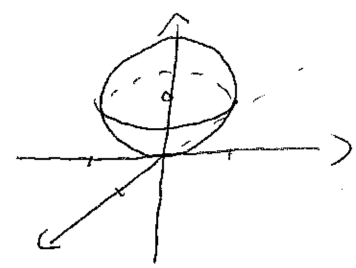
(b) [5pts.] Use your answer from part (a) to compute the integral of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ over the region inside the sphere of radius 1 and center $(0, 0, 1)$.

(a) $\rho^2 \leq 2\rho \cos\phi$

$\rho \leq 2\cos\phi$

Notice this is $x^2+y^2+(z^2-2z) \leq 0$

$x^2+y^2+(z-1)^2 \leq 1.$



(b) $\iiint_W \sqrt{x^2+y^2+z^2}\, dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{2\cos\phi} \rho(\rho^2\sin\phi)\, d\rho\, d\phi\, d\theta$

$= \left(\int_0^{2\pi} d\theta\right) \cdot \int_0^{\pi/2} \frac{1}{4}\rho^4 \sin\phi \Big|_0^{2\cos\phi} d\phi$

$= 2\pi \cdot \int_0^{\pi/2} \frac{1}{4}(2)^4 \cos^4\phi \sin\phi\, d\phi$

$= 2\pi \cdot 4 \cdot -\frac{1}{5}\cos^5\phi \Big|_0^{\pi/2}$

$= \frac{-8\pi}{5}[0-1]$

$= \frac{8\pi}{5}$